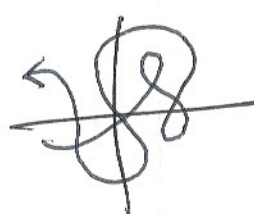


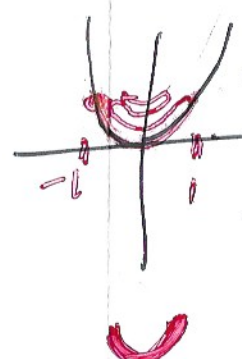
106 PARAMETRIC CURVES

PARAMETRIC EQUATIONS $\begin{cases} x = f(t) \\ y = g(t) \end{cases}$

t CALLED "THE PARAMETER"
↳ "TIME"

 $y = f(x)$
 $f(x, y) = 0$

$y = x^2$ DOESN'T TELL U WHICH PART?
DIRECTION?
"BACKTRACK"?
SPEED?

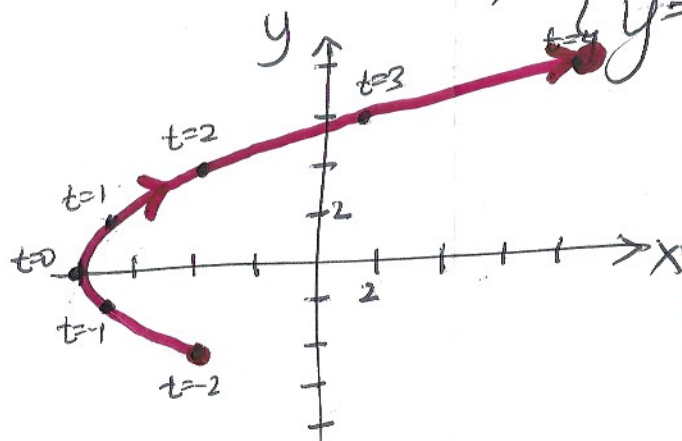


A PARAMETRIC CURVE IS THE LOCUS OF POINTS $(f(t), g(t))$ FOR t IN SOME CONTIGUOUS INTERVAL

SKETCH THE PARAMETRIC CURVE GIVEN BY $\begin{cases} x = t^2 - 8 \\ y = 2t \end{cases} t \in [-2, 4]$

BY PLOTTING POINTS

t	x	y	(x, y)
-2	-4	-4	$(-4, -4)$
-1	-7	-2	$(-7, -2)$
0	-8	0	$(-8, 0)$
1	-7	2	$(-7, 2)$
2	-4	4	$(-4, 4)$
3	-1	6	$(-1, 6)$
4	8	8	$(8, 8)$



LOOKS LIKE PART OF A PARABOLA, BUT IS IT REALLY?

ELIMINATING THE PARAMETER

METHOD 1: SOLVE FOR t IN ONE OF THE PARAMETRIC EQ'NS

$$x = t^2 - 8$$

$$y = 2t \rightarrow t = \frac{1}{2}y$$

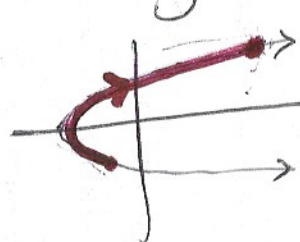
$$t \in [-2, 4]$$

SUBSTITUTE INTO THE EQ'N AND SIMPLIFY

$$x = \left(\frac{1}{2}y\right)^2 - 8$$

$$\star x = \frac{1}{4}y^2 - 8 \leftarrow \text{PARABOLA}$$

OPENS SIDeways TO THE RIGHT



$$\downarrow$$
$$x =$$

$$\downarrow$$
$$\frac{1}{4} > 0$$

(SIMILAR TO SOLVING EQUATIONS BY SUBSTITUTION)

ELIMINATE THE PARAMETER

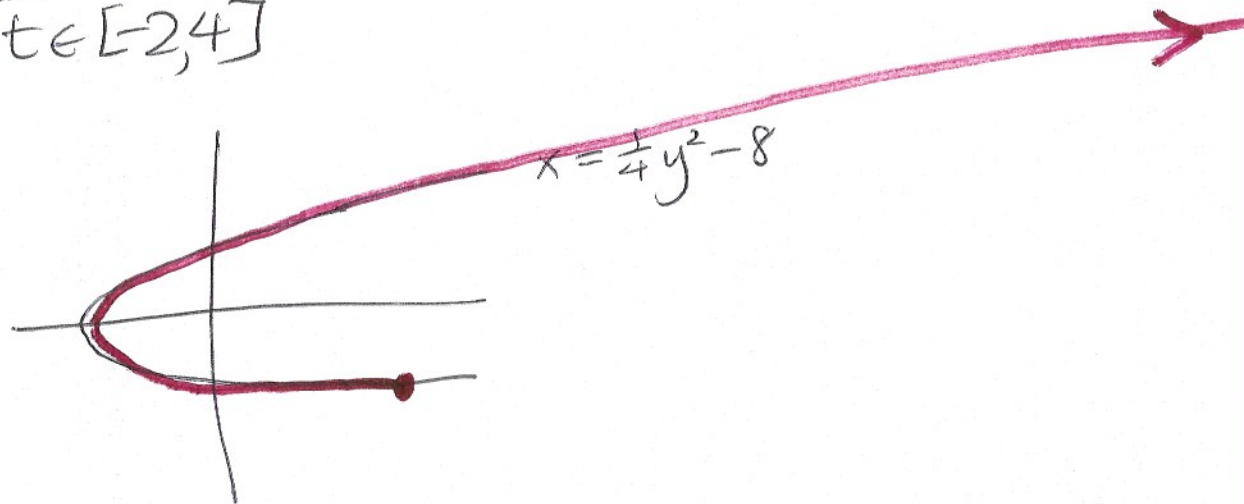
eg. $x = 4t^2 - 8, y = 4t, t \in [-2, 4]$

$$\downarrow$$
$$t = \frac{1}{4}y$$

$$x = 4\left(\frac{1}{4}y\right)^2 - 8$$

$$x = 4\left(\frac{1}{16}y^2\right) - 8$$

$$\star x = \frac{1}{4}y^2 - 8$$



10.6 PARAMETRIC CURVES

$$\begin{aligned}x &= f(t) \\ y &= g(t), \quad t \in [a, b]\end{aligned}$$

eg. $x = 4t^4 - 8, \quad t \in [-2, 4]$

$$y = 4t^2$$

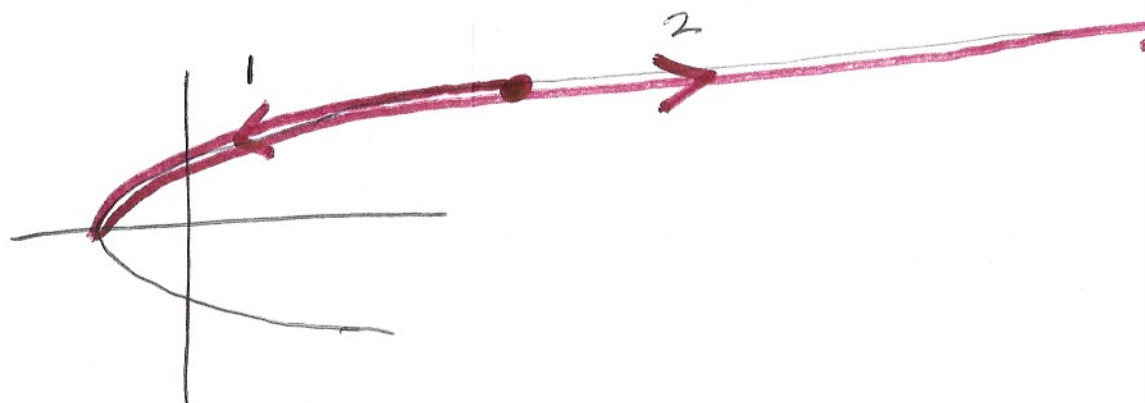


$$t^2 = \frac{1}{4}y$$

$$x = 4\left(\frac{1}{4}y\right)^2 - 8$$

★ $x = \frac{1}{4}y^2 - 8$

RECALL $t^4 = (t^2)^2$

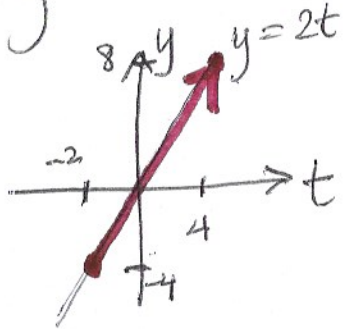


$$x = t^2 - 8$$

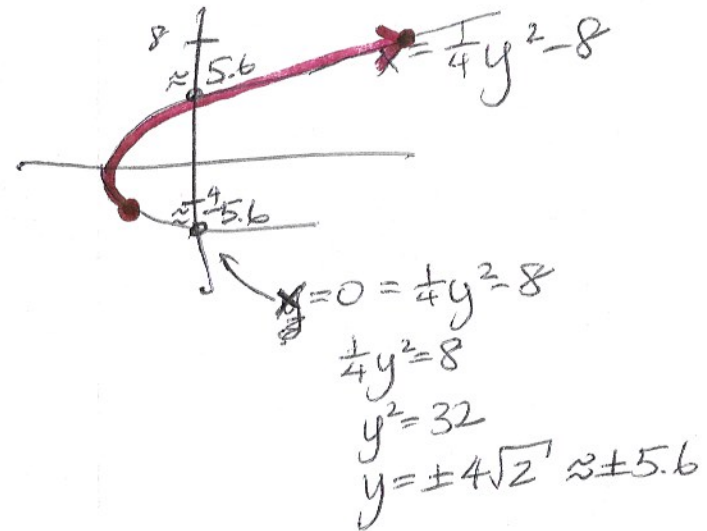
$$y = 2t, \quad t \in [-2, 4]$$

CONSIDER EASIER PARAMETRIC EQ'N

$$y = 2t$$



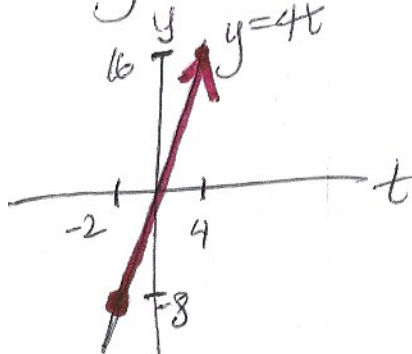
AS t GOES FROM -2 TO 4
 y -4 TO 8



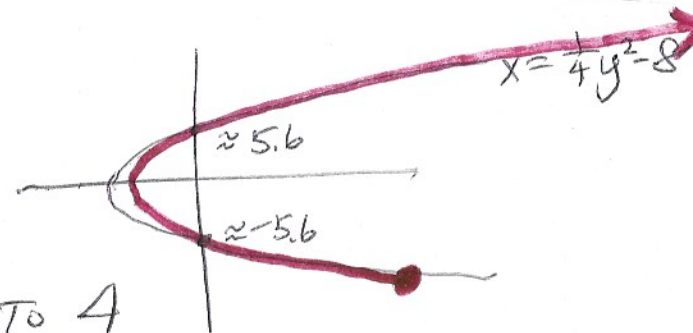
$$x = 4t^2 - 8$$

$$y = 4t, \quad t \in [-2, 4]$$

CONSIDER $y = 4t$



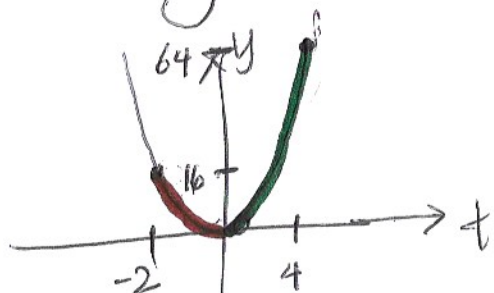
AS t GOES FROM -2 TO 4
 y -8 TO 16



$$x = 4t^4 - 8$$

$$y = 4t^2, \quad t \in [-2, 4]$$

CONSIDER $y = 4t^2$

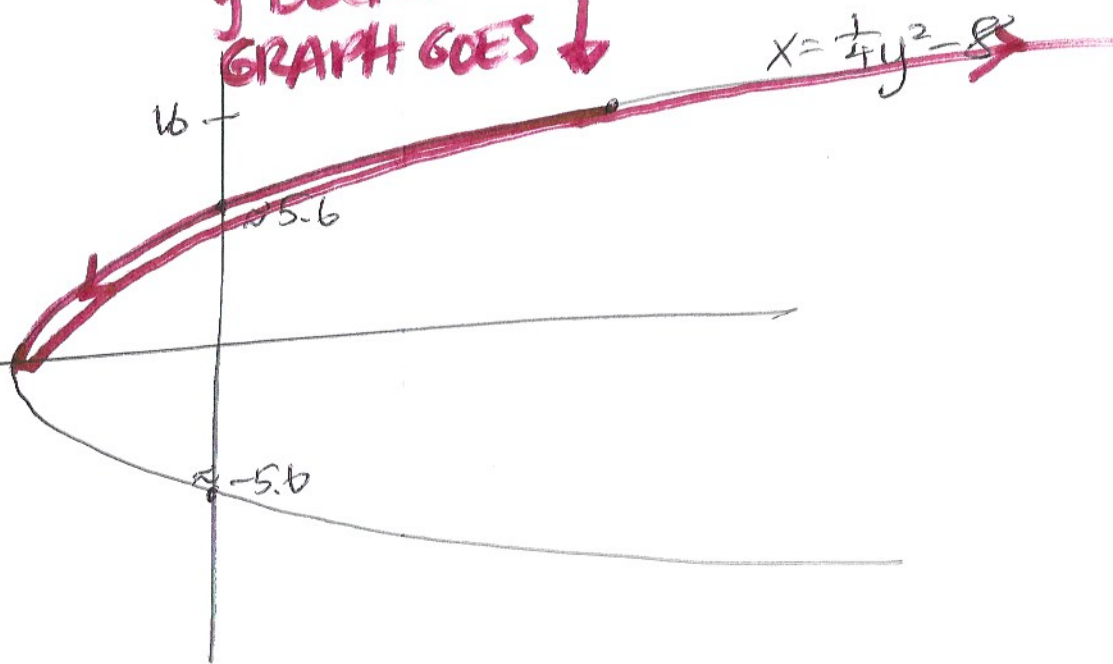


AS t GOES FROM -2 TO 0 TO 4
 y 16 to 0 to 64

y INCR
 GRAPH GOES $\uparrow$

y DECR
 GRAPH GOES $\downarrow$

INTERVAL ON WHICH y IS DECREASING INTERVAL ON WHICH y IS INCREASING



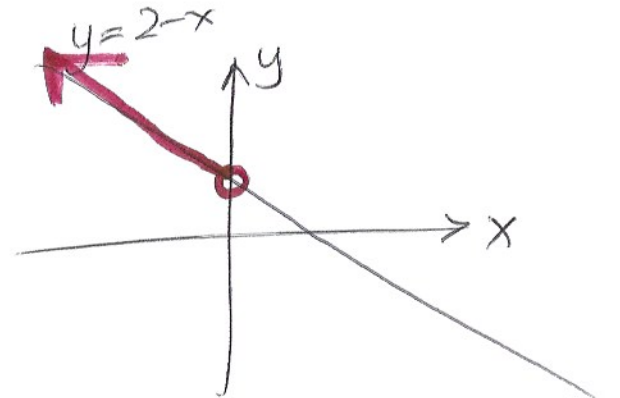
SKETCH $x = -e^t$ $t \in (-\infty, \infty)$ WITHOUT PLOTTING POINTS
 $y = 2 + e^t$

⑥ ELIM THE PARAM

$$x = -e^t \rightarrow e^t = -x$$

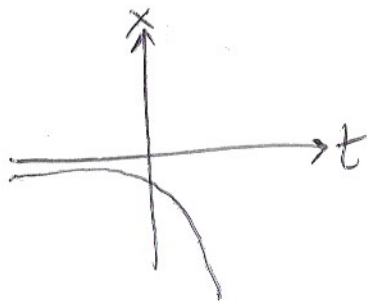
$$y = 2 + (-x)$$

$$y = 2 - x$$



① ANALYZE 1 EQ'N

$$x = -e^t$$



ALWAYS DECR

AS t GOES FROM $-\infty$ TO ∞
 x 0^- TO $-\infty$

$\uparrow$
 ≈ 0
 < 0

② LOCATE CORRESPONDING POINTS ON x, y CURVE